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ADAPTIVE KALMAN TYPE ESTIMATION APPLIED TO IMAGE PROCESSING*

by

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Abstract

Because of the stochastic and nonstationary nature of image processes, an adaptive estimation algorithm is proposed and evaluated for on-line filtering of an image scanned in a raster pattern. This algorithm combines a least squares parameter identification procedure with a two-dimensional reduced update Kalman filter. Results using an image with a 3 dB signal to noise ratio indicate that this adaptive algorithm is very effective for image restoration.

1. INTRODUCTION

Digital processing of images has in recent years become both economical and practical. Most sophisticated image processing is performed off-line on large machines because of the large memory and computational requirements of the largely used nonrecursive methods. In particular for the image estimation problem, classical nonrecursive techniques involve operations with large matrices and their inverses and are hence not suitable for real-time applications which might include:

1. Restoration of noisy images after reception on a low power transmission link.

2. Pictures arising from low light level imaging where background sensor noise significantly contributes to the output signal.

3. Reception of a decoded DPCM image which results from a maximum-likelihood decoding technique.

4. Processing of non-image two-dimensional (2-D) data for noise reduction prior to display in image format.

The use of digital computers for image estimation or as it is more popularly called image restoration started in the late 60's. The classical method due to Helstrom [1] was presented in 1967. Many variations on this original method were proposed [2], [3], [4]. However they all shared the need for manipulating large arrays with the attendant need for large general purpose computers.

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More recently various recursive algorithms have been derived which permit a more efficient realization [5], [8].

The methods discussed in [5] and [6] are however recursive in only one direction and therefore do not fully attain the efficiency of true 2-D recursive processing, where the recursion is in both dimensions. In [7] a method is presented for such recursive processing for scalar observations from a raster scan of a 2-D data set. Recent results however [9], point out errors in the derivation in [7] casting doubt on the validity of the entire method.

More recently Woods and Radewan [10] have proposed the reduced update Kalman filter as a means for alleviating the computational problems which had precluded the implementation of 2-D estimation algorithms. This filter was shown to be optimum in the sense of minimizing the post update mean square error subject to the constraint of updating only the "nearby" previously processed picture elements.

It should be noted however that the reduced update Kalman filter and other proposed pixel estimation procedures require an autoregressive type model whose coefficients must be determined from some available set of similar images [11]. Since such sample images are not always available and since images in general are nonstationary and not necessarily true autoregressive processes, it is interesting to consider the implementation of an apaptive 2-D estimator. Such an adaptive estimator (analogous to an adaptive controller) would consist of an identification algorithm which calibrates the picture model coefficients used by the Kalman estimator. One such configuration recently proposed by Keshavan and Srinath combines the identification of a 2-D interpolative model with a suboptimal estimation algorithm [12]. Although their resulting image enhancement was rather impressive, their proposed procedure does not appear suitable

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for on-line implementation with a raster type scanning system. Furthermore, their proposed model has a limited number of variable coefficients, and the identification does not take into account any nonstationarity in the image process.

Consequently towards the goal of developing an adaptive estimation algorithm suitable for on-line implementation on a raster scan of nonstationary images, this paper discusses the use of various parameter identification algorithms for adapting the model used by the reduced update filter developed in [10]. In general it can be concluded that adaptation of the model results in a significant reduction in the rms error between the true and estimated pixel intensities.

In order that the reader obtains a complete description of the algorithms, the reduced update filter is summarized in Section 2 followed by a description of the parameter identification procedures in Section 3. Experimental procedures are then described in Section 4, with results and conclusions presented in Sections 5 and 6 respectively.

2. REDUCED UPDATE KALMAN FILTER

In one dimension, the Kalman filter offers an attractive solution to the linear filtering and prediction problem. The extension of one-dimensional Kalman filtering to two dimensions requires not only a suitable 2-D recursive model but also an enormous amount of data storage and transfer due to the high dimension of the resulting state vector. Hence a straightforward extension is of limited success, and thus it becomes desirable to consider computationally effective approximations. Here one such approximation, the 2-D reduced update Kalman filter as presented in [10] is reviewed.

To illustrate this approach, consider the scanning of a discrete 2-D field on an NxN regularly spaced lattice. Since the scanning operation does not qualitatively affect the results, a raster scan is assumed.

To be considered is a signal which is Markovian and given by a nonsymmetric half-plane (NSHP) recursive model.

$$s(m,n) = \sum_{\mathcal{R}_{\oplus+}} c_{k\ell}\, s(m-k,n-\ell) + w(m,n) \qquad (1)$$

where $w(m,n)$ is a white Gaussian noise field and $\mathcal{R}_{\oplus+}$ is an NSHP, i.e. $[m \geq 0,\ n \geq 0] \cup [m<0,\ n>0]$. It is further assumed that this model is (MxM)th order.

The observation model is

$$r(m,n) = s(m,n) + v(m,n) \qquad (2)$$

where $v(m,n)$ is a white Gaussian source. Using the scanning operation the 2-D problem is transformed into an equivalent 1-D problem by defining a state vector of M(N+1) components:

$$\underline{s}(m,n) = [s(m,n),\ s(m-1,n),\ldots,s(1,n);\ s(N,n-1)\ldots s(1,n-1);\ldots;s(N,n-M),\ldots,s(m-M,n-M]^T$$

then (1) and (2) can be put into the form,

$$\underline{s}(m,n) = \underline{\underline{C}}\ \underline{s}(m-1,n) + \underline{w}(m,n), \qquad (3)$$

$$r(m,n) = \underline{H}\ \underline{s}(m,n) + v(m,n) \qquad (4)$$

Thus, the Kalman equations with the above interpretation of the $\underline{s}$ vector can be immediately written down. The difficulty with these equations is the amount of computation and memory requirements associated with them. By limiting the update process to only those elements "near" the "present" point, the computation can be greatly reduced. The resulting reduced update Kalman filter equations can be written in scalar form as given below. For details see [10]. In these equations, the superscript indicates the step in the filtering, while the argument represent the position of the data on the NxN grid. Subscript "a" and "b" indicate "after" and "before" updating, respectively.

State Prediction and Update:

$$\hat{s}_b^{(m,n)}(m,n) = \sum_{k,\ell} c_{k\ell}\,\hat{s}_a^{(m-1,n)}(m-k,n-\ell) \qquad (5)$$

$$\hat{s}_a^{(m,n)}(i,j) = \hat{s}_b^{(m,n)}(i,j) + K^{(m,n)}(m-i,n-j)\cdot [r(m,n) - \hat{s}^{(m,n)}(m,n)],\ (i,j)\in \mathcal{R}_{\oplus+}^{m,n} \qquad (6)$$

Error Covariance and Gain:

$$R_b^{(m,n)}(m,n;k,\ell) = \sum_{o,p} c_{op} R_a^{(m-1,n)}(m-o,n-p;k,\ell),\quad (k,\ell)\ \epsilon\ \mathcal{S}_{\oplus+}^{(m,n)} \qquad (7)$$

$$R_b^{(k,\ell)}(m,n;m,n) = \sum_{k\ell} c_{k\ell} R_b^{(m,n)}(m,n;m-k,n-\ell) + \sigma_w^2 \qquad (8)$$

where $\mathcal{S}_{\oplus+}^{(m,n)}$ is the support of the state vector $\underline{s}(m,n)$.

$$R_a^{(m,n)}(i,j;k,\ell) = R_b^{(m,n)}(i,j;k,\ell) - K^{(m,n)}(m-i,n-j)\cdot R_b^{(m,n)}(m,n;k,\ell),\quad (i,j)\in\mathcal{R}_{\oplus+}^{(m,n)};(k,\ell)\ \epsilon\ \mathcal{S}_{\oplus+}^{(m,n)} \qquad (9)$$

$$K^{(m,n)}(i,j) = R_b^{(m,n)}(m,n;i,j)/[R_b^{(m,n)}(m,n;m,n) + \sigma_v^2],\ (i,j)\ \epsilon\ \mathcal{R}_{\oplus+} \qquad (10)$$

Further reduction in computation can be obtained by computing (7) and (9) in a fixed size region, smaller than $\mathcal{S}_{\oplus+}^{(m,n)}$. Such a region will be written as $\mathcal{T}_{\oplus+}^{(m,n)}$ (see Figure 1).

3. IDENTIFICATION ALGORITHMS

The implementation of the reduced update Kalman filter algorithm (5-10) requires the knowledge of the $c_{k\ell}$'s in (5). In the following, various algorithms are sugested for identification of these unknown parameters. To this effect, if the $c_{k\ell}$'s are ordered into a column vector $\underline{c}$, (1) can be rewritten as:

$$s(m,n) = \underline{c}^T \underline{s}_1(m,n) + w(m,n), \tag{11}$$

where $\underline{s}_1$ is the portion of the state vector $\underline{s}$ in the model's active memory, i.e., the pseudo-state vector [10].

Images are, in general, non-homogeneous and hence the elements of $\underline{c}$ are spatially variant. Ideally, then, a $\underline{c}$ vector should be found for each pixel (m,n).

Thus at selected intervals an estimate $\underline{c}$ for the coefficient vector at point (m,n) may be determined so as to minimize the general weighted least squares index:

$$J_{m,n} = \sum_{i,j\in\Omega_{m,n}} (r(i,j) - \underline{c}^T \underline{r}_1(i,j))^2 \cdot W(i,j;m,n) \tag{12}$$

where $\underline{r}_1$ is the vector of measurements (from eq. (2)) of $\underline{s}_1$ and $\Omega_{m,n}$ is the data history over which the index is to be minimized at point (m,n), and $W(i,j;m,n)$ is a fading memory weighting factor.

The variance of the plant noise ($w(m,n)$ of (1)) was estimated using:

$$\sigma_w^2(m,n) = \frac{\gamma}{N_{m,n}} \sum_{i,j\in\Omega_{m,n}} (r(i,j) - \underline{c}^T \underline{r}_1(i,j))^2 \tag{13}$$

where $N_{m,n}$ is the number of points in the data history and γ is an adjustment constant chosen empirically to account for the error in using r and $\underline{r}_1$ rather than s and $\underline{s}_1$. Although minimization of (12) is known to yield biased estimates it was felt that in view of earlier work [13], such an approach might yield acceptable results. Present efforts are however being devoted to the study of techniques, such as that proposed by Kotob and Kaufman [14], which take into account the bias caused by the interaction of the noise in both r and $\underline{r}_1$.

For purposes of comparison, the following identification algorithms were tested:

1. Infinite memory, general least squares, i.e., $W(i,j;m,n)=1$, $\Omega_{m,n} = [i,j \mid 1\le i\le m,\ 1\le j\le N-1;\ j=N,\ 1\le i\le m-1]$. Recursive updates were used.

2. Infinite memory, general least squares fit over distinct square block segments of the picture of dimension KxK, i.e., $W(i,j;m,n)=1$. $\Omega_{m,n} = [i,j \mid i,j\in \text{selected block}]$

   In this case the estimate was computed after receipt of the entire block by:

$$\hat{\underline{c}} = [\sum_{j=1}^{K} \sum_{i=1}^{K} \underline{r}_1(i,j)\, \underline{r}_1^T(i,j)]^{-1} \cdot [\sum_{i=1}^{K} \sum_{j=1}^{K} \underline{r}_1(i,j)\, r(i,j)]. \tag{14}$$

   As a variant to help reduce the effects of bias, the measurement noise variance times the unit matrix was subtracted from the first summation prior to inversion.

3. Fixed memory, general least squares fit, i.e.,

   $W(i,j;m,n) = 1.$

   $\Omega_{m,n} = [i,j \mid i,j \in$ a region similar in form to $\mathcal{R}_{\theta+}$ shown in Figure 1b]. The memory is said to be of order (LxL) if the numbered pixels to the left, right, and above pixel (m,n) is equal to L. Because of problems inherent to the development of a recursive implementation, a non-recursive algorithm was used.

4. Fading memory least squares fit, i.e.,

   $W(i,j;m,n) = \exp(-\alpha|m-i|-\alpha|n-j|)$.

   $\Omega_{m,n}$ was the same as for case 3 in order to simplify computation.

4. EXPERIMENTAL PROCEDURES

The noise free image as shown in Figure 2, is a 128 x 128 image data field. Additive measurement noise $w(m,n)$ was simulated using a Gaussian white noise generating subroutine. For simulation purposes, a 3 db signal to noise ratio was used, with noise variance equal to 1442 giving the picture shown in Fig. 3. These parameter values and associated plant noise were passed on to the reduced update Kalman filter. From previous experiments, the required $\mathcal{R}_{\theta+}$ and $\mathcal{T}_{\theta+}$ regions were chosen as shown in Fig. 4. Boundary conditions for eqns. (7), (8), and (9) were assumed to be diagonal while those for (5) and (6) were assumed to be zero. Though the parameter and gain values were changed across the boundary of each block in the filter, no detectable edge effects were noted in the estimated image.

The estimated image was then compared with the noise-free data, and evaluation was based upon the estimation index,

$$J = \frac{1}{\mathcal{N}} \sum_i \sum_j (\hat{s}(i,j) - s(i,j))^2 \tag{15}$$

where $s(i,j)$ is the true pixel intensity and $\hat{s}(i,j)$ is its reduced update estimate based upon the identified model and $\mathcal{N}$ is the total number of pixels. Also considered was the identification residual index,

$$I = \frac{1}{\mathcal{N}} \sum_m \sum_n (r(m,n) - \hat{\underline{c}}^T(m,n)\underline{r}_1(m,n))^2 \tag{16}$$

computed at point (i,j) for $\underline{c}$.

5. RESULTS

Initially in order to evaluate the various identification algorithms only two (32 x 32) blocks in the mouth-chin area of Fig. 3 were processed. Results summarized in Table 1 show that:

- o All algorithms reduced the estimation "variance" J well below the observation noise variance of 1442.
- o Infinite memory recursive least squares is not satisfactory because of the nonstationary nature of the image.
- o A data history of order larger than (3 x 3) is needed to reduce the effects of measurement noise.

Other experiments (not summarized in Table 1) showed that:

- o A 10-20% improvement resulted from using a $(2 \times 2)^{th}$ order model with 12 coefficients rather than the $(1 \times 1)^{th}$ order model.
- o Estimation results (J) could be improved significantly (30%) in many cases by using a smaller value for σ_w^2 (Eq. 13). The adjustment factor γ was found empirically to be 0.3 for all cases.

Comparison of Fig. 5, which corresponds to an infinite memory identification, with Fig. 6, which corresponds to block processing, shows the need for a finite memory size if rapidly changing details (i.e., teeth) are to be processed.

Algorithm	Processing Characteristics	J (Eq. 15)	I (Eq. 16)
Infinite memory-recursive least squares	Identification at every point	684	1953
$\gamma = .3$	Identification at every third point	655	1987
Block Processing	No bias correction	310	1927
$\gamma = .3$	0.8 x noise variance - removal from first summation in Eq. 14	298	2010
Fixed Memory $\gamma = .3$	(3x3) data history $\alpha = 1$	537	238
	5 x 5 data history $\alpha = 0.1$	306	421

Table 1 Processing Results Using a (1 x 1)th Order Model Over 2 (32 x 32) Blocks

6. CONCLUSION

Basedupon the preceding results, it can be concluded that a fixed memory least squares parameter identifier used for adaptation of the model in a reduced update two-dimensional Kalman filter is very effective for image recovery. Under study at the present time are efforts directed towards:

- . Removing the bias in the parameter estimates.
- . The study of simultaneous parameter and pixel estimation (e.g., via extended Kalman filter and/or quasilinearization).
- . Implementation of a fixed memory filter through the use of a stochastic dynamic parameter model.

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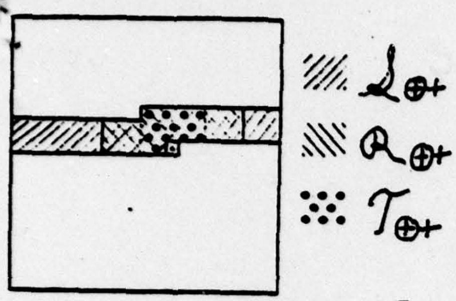

Fig. 1 Specifications of $\mathcal{S}, \mathcal{R}, \mathcal{T}$ Used to Define Updated Regions

Fig. 2 Noise Free Image

Fig. 3 Image with Noise (SNR = 3 db)

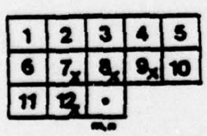

Fig. 4a $\mathcal{R}$ Region (x's Denote Appropriate Pixels for (1 x 1)th order Model)

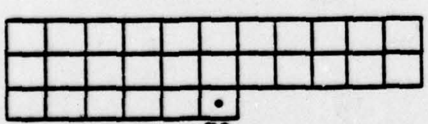

Fig. 4b $\mathcal{T}_{\oplus}$ Region

Fig. 5 Results Using an Infinite Memory

Fig. 6 Results Using Block Processing